Linear Algebra I

17/12/2018, Monday, 15:00 – 17:00

You are **NOT** allowed to use any type of calculators.

---

1 Linear systems of equations (5 + 10 + 5 + 5 = 25 pts)

A company produces 3 types of products: **A**, **B**, and **C**. For a type **A** product, it takes 20 minutes to assemble, 4 minutes to test, and 4 minutes to pack. A type **B** product requires 24 minutes to assemble, 5 minutes to test, and 4 minutes to pack. Finally, a type **C** product requires 12 minutes to assemble, 3 minutes to test, and 3 minutes to pack. Given that the company can afford 3120 minutes per day for assembling, 680 minutes for testing, and 640 minutes for packing, we want to find how many of each kind can be produced in a day.

(a) Let x, y, and z be number of, respectively, type **A**, **B**, and **C**, products that produced each day. Find the linear equations relating x, y, z and write down the augmented matrix for those linear equations.

(b) By performing elementary row operations, put the augmented matrix into **reduced** row echelon form.

(c) Determine whether the equation system is consistent or inconsistent.

(d) Determine the solution set.

2 Determinants (15 pts)

Let a, b, c, and d be real numbers. Find the determinant of the matrix

$$\begin{bmatrix} a^2-1 & a-1 & a-1 & a-1 \\ a+1 & b^2 & b & b \\ a+1 & b+2 & c^2+1 & c+1 \\ a+1 & b+2 & c+3 & d^2+2 \end{bmatrix}.$$

3 Partitioned matrices (15 + 5 = 20 pts)

Let A be $n \times n$ nonsingular matrix. Consider the matrix

$$M = \begin{bmatrix} A & A & 0 \\ A & I & A \\ 0 & A & A \end{bmatrix}$$

where I and 0, respectively, denote the $n \times n$ identity and zero matrices.

(a) Show that the matrix M is nonsingular if and only if $I - 2A$ is nonsingular.

(b) Let $A = I$ and find the inverse of M.

4 Vector spaces (15 + 15 = 30 pts)

Consider the vector space P_4. Let

$$S = \{p \in P_4 \mid p(0) = p(1) = p(2)\}.$$

(a) Show that S is a subspace.

(b) Find a basis for S and determine its dimension.

---

10 pts free